Control Problems during Start-Up of Solid Core Nuclear Rockets

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Nomenclature

A_c = heat-transfer surface area per unit length of core
B^2 = buckling
C/U = carbon to fuel atom ratio in core
c_p = heat capacity
D = diffusion coefficient
d = coolant channel diameter
h = heat-transfer coefficient
k = thermal conductivity
L = core length
Nu = Nusselt number evaluated at T_{Hb}, hd/k
P = power density
Pr = Prandtl number evaluated at T_{Hb}, $c_p\mu/k$
Q = slowing-down density at lethargy u due to hydrogen only
q = slowing-down density u due to carbon only
Re = Reynolds number evaluated at T_{Hb}, $dv\rho/k$
T = temperature
t = time, sec
V_c = volume of core per unit length
X = axial position in core
ϕ = neutron flux
u = unit lethargy
v = gas velocity
ρ = density
$\dot{w}$ = hydrogen flow rate
Σ_a = macroscopic absorption cross section
Σ_s = macroscopic scattering cross section
μ = viscosity
ζ = average lethargy change per collision for scattering by carbon

Subscripts and Superscripts

c = reactor core
H = hydrogen gas
C = carbon
b = bulk conditions

THIS note investigates some of the nucleonic problems encountered in starting large graphite-moderated nuclear rocket reactors fueled with uranium-235. Primary emphasis is placed on obtaining the reactivity variations during start-up induced by changes in core temperature and hydrogen density.

For a specified void fraction and coolant channel diameter, large thrust can only be achieved by using a physically large core. Using the rule-of-thumb for nuclear rockets[1] that

$$\text{thrust (lb)} = 50 \times \text{power (Mw)} \tag{1}$$

and assuming a power density[2] of 2 Mw/l, it can be shown that, to produce a thrust of 10^6 lb, a reactor core with dimensions of several meters is required. However, as the core size is increased, the atom ratio of carbon to fuel (C/U) in the core must be increased to keep the reactivity from becoming excessive, and this, in turn, causes the neutron spectrum to become more thermal. The reactivity in a thermal reactor usually decreases with increasing core temperature. Since the temperature of a nuclear rocket reactor core may increase by as much as 2000°K during a 30-sec start-up,[3] serious control problems could arise in large thermal reactors. Moreover, the control problem during start-up is made more complex by the presence of hydrogen, which has a large positive reactivity value due to its excellent neutron moderating characteristics.

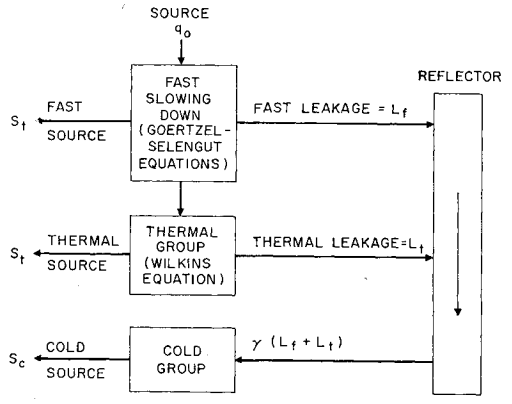

Fig. 1 Schematic diagram of reactor physics model.

Reactivity Variation Due to Temperature and Hydrogen Density

Homogeneous changes in both hydrogen density and core temperature in a nuclear rocket reactor have an appreciable influence on the neutron energy distribution, but do not significantly alter the spatial neutron distribution.[4] Therefore, to compute reactivity variations during start-up, it appears appropriate to use a nucleonics model that treats the neutron energy distribution in detail, but only approximately considers the spatial distribution (Fig. 1). For a given source q_0, the slowing down spectrum is computed to obtain the fast fission source S_f, the fast leakage, and thermal slowing-down density. The thermal spectrum is then computed to find the thermal fission source S_t and thermal leakage. Assuming that a certain fraction γ of the leakage neutrons are reflected into the core and cause fission S_c, the multiplication constant k is

$$k = (S_f + S_t + S_c)/q_0 \tag{2}$$

The Goertzel-Selengut equations,[5] which rigorously treat the slowing down by hydrogen, are employed to compute the neutron energy spectrum from 1 Mev to 1.44 ev. In this region, all thermal motion of the moderator is neglected. Assuming that $\nabla^2\phi(u) = -B^2(u)$, the equations are

$$dq/du = Q - q[DB^2 + \Sigma_a + \Sigma_s^{\mathrm{H}}]/\zeta\Sigma_s^{\mathrm{C}} \tag{3}$$

$$d\eta/du = -Q + q\Sigma_s^{\mathrm{H}}/\zeta\Sigma_s^{\mathrm{C}} \tag{4}$$

They are solved for a given reactor core by a numerical integration method using 55 lethargy groups. The cross sections for each group were taken from the GAM-I Library Tape.[6] All neutrons that thermalize to 1.44 ev are assumed to enter the thermal group (Fig. 1). For a specified core temperature T_c, the thermal neutron distribution is computed from the Wilkins equation.[7]§ Knowing the thermal distribution, the nuclear properties for the thermal group are computed by averaging the energy-dependent properties over the thermal flux distribution.

In both the slowing-down and thermal-energy ranges, the neutron leakage from the core is computed from the assumption that $D(u)\nabla^2\phi(u) = -D(u)B^2\phi(u)$. The leakage neutrons are assumed to be thermalized in the cold reflector. A fraction of these cold neutrons are then reflected into the core where they are either captured or cause fission. For a given

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§ The Wilkins equation entails numerous approximations.[7] Perhaps the most restrictive assumption is that the absorption cross section must vary as (neutron velocity)$^{-1}$.

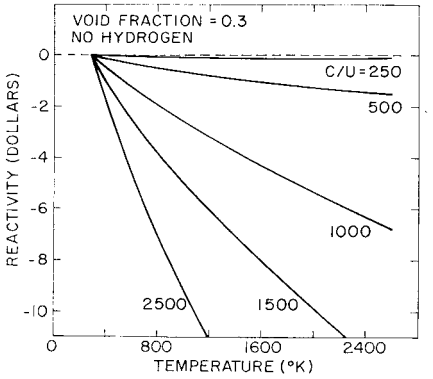

Fig. 2 Reactivity variation for homogeneous changes in core temperature.

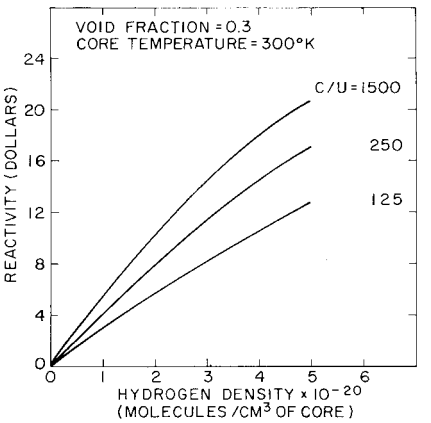

Fig. 3 Reactivity variation for uniform changes in hydrogen density.

reflector, the fraction returning into the core (the albedo) is determined from a space-dependent nucleonics model with three energy groups.[4] Because the model has no spatial dependence, it should not be expected to yield precise values for critical size and mass; however, the reactivity variations brought about by changes in hydrogen density and core temperature should be accurately reflected.

All calculations are for a beryllium-reflected core with a void fraction of 0.3. For each carbon-to-fuel ratio, the core size (or buckling) is determined so that the reactor is just critical at 300°K with no H_2.

Figure 2 shows the reactivity as a function of core temperature for various C/U ratios. Cores with intermediate neutron spectrums (C/U $\cong$ 250) are rather insensitive to temperature changes, because they have very few neutrons in the thermal energy range. Larger cores (higher C/U) are more sensitive to temperature changes because more neutrons reach thermal equilibrium. Note that, for thermal cores (C/U $\cong$ 2500), the total reactivity change for a temperature increase from 300° to 2000°K during start-up is of the order of 14 dollars.

Figure 3 shows the effect of hydrogen density. Because of its ability to reduce the fast leakage by moderating the neutrons before they have sufficient time to diffuse out of the core, one would expect that the addition of hydrogen to a reactor core would increase the reactivity. For small cores (C/U = 125), the effect of hydrogen on reactivity is relatively small, because few neutrons reach the thermal energy range with or without it. As the core size increases, the effect increases because hydrogen scatters additional neutrons into the important thermal region. However, for C/U $\gg$ 1500, the hydrogen-reactivity worth would drop, because the neutron spectrum is practically thermalized by graphite moderation alone. Note that, for a hydrogen at 1000°K and 70 atm ("typical" conditions), ρ is about 1.5×10^{20} molecules/cm^3 of core. For thermal cores, the reactivity change for this density is not as significant as that due to temperature variations.

Reactivity Variations during Start-Up

The reactor power in the core and the propellant flow rate through the core are assumed to increase linearly with time during start-up. The axially dependent core temperature and hydrogen density are then determined by a heat-transfer model. Using average values of T_c and ρ_H, the reactivity variation during start-up is obtained from the foregoing nucleonics model. The following assumptions are made: 1) spatial power distribution does not vary during start-up, 2) only one dimension (axial) is necessary, 3) hydrogen "residence time" in the core is much less than the start-up period, 4) Mach number of the coolant is $\ll$1.0, and 5) the heat-transfer coefficient h can be approximated by[8]

$$Nu = 0.045\, Re^{0.8}\, Pr^{0.4}(T_c/T_H)^{0.55}(L/d)^{-0.15} \quad (5)$$

where the thermal properties of H_2 at the bulk gas conditions (ρ_{Hb}, T_{Hb}) are computed from power-series expressions.

With these assumptions, the time-dependent heat-balance equations take the forms

$$\rho_c V_c c_{pc}(dT_c/dt) = V_c P(X,t) - A_c h(X,T_c)[T_c(X,t) - T_H(X,t)] \quad (6)$$

$$\dot{w}(t) c_{pH}(dT_H/dX) = A_c h(X,T_H)[T_c(X,t - T_H(X,t)] \quad (7)$$

To obtain numerical results, these equations are reformulated into difference equations and solved for a specified P and $\dot{w}$.

Figure 4 shows the reactivity variation during start-up for various C/U ratios. The important parameters used are initial T_c = 300°K, inlet T_H = 140°K, exit T_H = 2500°K at 70 atm, core void fraction = 0.3, and d = 0.25 cm. The initial dip in the curves is caused by the rapid increase in T_c, which reaches its terminal value in roughly half the start-up time. The larger the core, the more pronounced the temperature dip. The linear increase of ρ_H with t explains the smooth reactivity increase after the initial temperature dip. The shapes of these curves would be altered if the power and flow rate were not linear functions of time during start-up; however, any start-up procedure in a thermal rocket reactor will induce very significant reactivity variations due to the large temperature coefficient.

Concluding Remarks

Start-up of large nuclear rocket reactors can present serious control problems. The magnitude of reactivity change in-

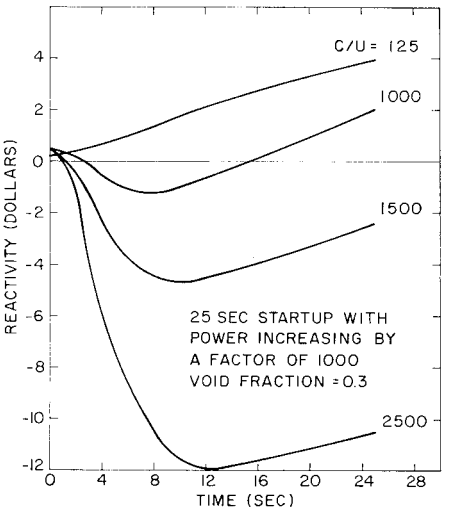

Fig. 4 Reactivity variation during start-up.

duced by changes in core temperature and hydrogen density would require a sizeable control system, and the rapid rate of change would require both a sensitive and a fast-acting control system. These stipulations may be difficult to fulfill.

There are ways to reduce the reactivity variation during start-up. An obvious method is to place a nuclear poison in the core to harden the thermal spectrum, which in turn reduces the change in nuclear properties induced by temperature variations. However, this procedure has the adverse side effect of reducing the external control rod worth by reducing the neutron leakage.

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Rocket Motor with Electric Acceleration in the Throat

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PRESENT rocket motors seem to be inadequate for long space missions because of poor mass utilization due to limited exhaust velocity (low specific impulse $I_{sp} < 500$ sec). One can obtain much better mass utilization by using the concept of electric propulsion (I_{sp} of the order of 10,000). However, for intermediate specific impulses around 1000 to 2000, the combination of both seems to be adequate. The chemical process in the combustion chamber will provide high enough electric conductivity by adding seeding substance (1% potassium or cesium). The gas is accelerated to the sonic speed in the convergent nozzle (Fig. 1), then, in the straight channel with segmented electrodes and the normal magnetic and applied electric fields, it is accelerated to supersonic speed with relatively small pressure drop. Final expansion occurs in the divergent nozzle. The location of the electric accelerator in the throat offers the following advantages: 1) high enough temperature and therefore sufficient electric conductivity, and 2) minimum spacing between magnetic field coils. The present scheme requires a separate energy source that must be a nuclear power generator.

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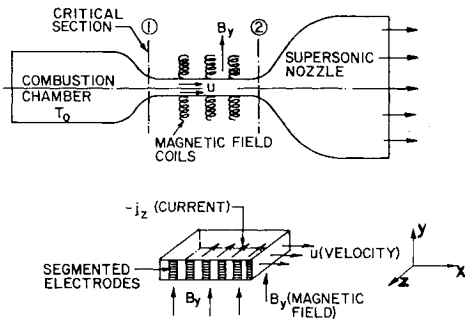

Fig. 1 Rocket motor with electric acceleration in the throat.

Fundamental Equations

Assuming one-dimensional flow at low magnetic Reynolds number one obtains

Conservation of Mass

$$\rho u = \rho_1 u_1 = \text{const} \tag{1}$$

where u denotes velocity, ρ is density, and subscript 1 denotes reference parameters.

Conservation of Momentum

$$\rho u(du/dx) + (dp/dx) = -j_z B_y \tag{2}$$

where the right-hand side represents Lorentz force.

Conservation of Energy

$$\rho T \frac{DS}{Dt} = \frac{pu}{\gamma - 1}\left(\frac{1}{p}\frac{dp}{dx} - \frac{\gamma}{\rho}\frac{d\rho}{dx}\right) = \frac{j_z^2}{\sigma} \tag{3}$$

or

$$\gamma p \frac{du}{dx} + u\frac{dp}{dx} = (\gamma - 1)\frac{j_z^2}{\sigma} \tag{3'}$$

The terms j_z^2/σ represent joule heating (i_z is current density, and σ is electric conductivity).

Ohm's Law

$$j_z = -\sigma(E - uB_y) \tag{4}$$

where E_z is an applied electric field, and B_y is the magnetic field.

The following dimensionless variables are introduced:

$$M_1 = \frac{u_1}{a_1} \qquad \bar{u} = \frac{u - u_1}{a_1} \qquad \bar{p} = \frac{p}{p_1} \qquad \bar{a} = \frac{a}{a_1}$$

(velocity of sound ratio)

$$\bar{j} = \frac{j_z}{\sigma B_y a_1} \qquad \Lambda = \frac{E_z}{B_y a_1} \qquad \xi = \frac{a_1 B_y^2 \int \sigma dx}{p_1}$$

The introduction of ξ makes the foregoing equations independent of the conductivity σ and its variation with temperature. However, the effective physical distance depends on σ.

Equations (2–4) can be written in the forms

$$\gamma M_1(d\bar{u}/d\xi) + (d\bar{p}/d\xi) = -\bar{j} \tag{5}$$

$$\gamma\bar{p}\frac{d\bar{u}}{d\xi} + (M_1 + \bar{u})\frac{d\bar{p}}{d\xi} = (\gamma - 1)\bar{j}^2 \tag{6}$$

$$\bar{j} = M_1 + \bar{u} - \Lambda \tag{7}$$

From Eq. (1) the velocity of sound is

$$a = \{\bar{p}[1 + (u/M_1)]\}^{1/2} \tag{8}$$

From Eqs. (5) and (6) one gets

$$\frac{d\bar{u}}{d\xi} = \frac{-(M_1 + \bar{u})\bar{j} + (\gamma - 1)\bar{j}^2}{\gamma M_1(M_1 + \bar{u}) - \gamma\bar{p}} \tag{9}$$